Acoustic Resonance

Description of Activity

Stringed acoustic instruments, such as the guitar and ukulele, are capable of producing some of the most amazing classical and popular music and are reasonably accessible to learners of all ages. In appreciation of acoustic instruments, musical theory, and the fields of mathematics and science, this activity will explore how the hollow body of a guitar/ukulele works as an acoustic resonator, as well as what these instruments have in common with the Helmholtz resonator.

Learning Objectives:

1. Students will learn about the guitar/ukulele body as an acoustic resonator.
2. Students will be introduced to the physics and math concepts of the Helmholtz resonator.
3. Students will measure and calculate the volume of the body of an acoustic instrument (irregular shape) and the radius and area of the sound hole.
4. Students will apply the partial differential equation satisfied by the Helmholtz resonator to determine the instrument's frequency.

Standards:

Common Core Math Standards and Next Generation Science Standards aligned with this activity:

CCSS.Math.Content.HSA-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step.
CCSS.Math.Content.HSA-SSE.A.1b Interpret complicated expressions by viewing one or more of their parts as a single entity.
CCSS.Math.Content.HSG.MG.A.1 Use geometric shapes, measures, and properties to describe objects.
CCSS.Math.Content.HSG.MG.A.3 Apply geometric methods to solve design problems.
CCSS.Math.Practice.MP1 Make sense of problems and persevere in solving them.
CCSS.Math.Practice.MP4 Model with mathematics.
CCSS.Math.Practice.MP5 Use appropriate tools strategically.
CCSS.Math.Practice.MP6 Attend to precision.
NGSS-HS-ETS1-3 Evaluate a solution to a complex real-world problem based on prioritized criteria and trade-offs that account for a range of constraints
NGSS-HS-PS4-1 Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling in various media.

Materials Required:

- Acoustic instruments – guitar/ukulele (ideally, one per each student, student pair, or small group)
- Measurement tools (calipers, tape measure)
- Masking tape
- Calculator
- Optional - one water bottle per student (for experimentation along with the video)

Safety:

- N/A

References:

- Common Core State Standards Initiative. (2012). Retrieved from: http://www.corestandards.org/Math
- French, Mark. (2016). *Helmholtz resonator*. Brain Waves.avi [You Tube video]. Retrieved from: https://youtu.be/s2ocI9AV0KA
- French, R.M. (2005). *A pop bottle as a helmholtz resonator.* Experimental techniques: My favorite experiment series. Retrieved from: http://web.ics.purdue.edu/~rmfrench/Downloads/Pop_Bottle.pdf
- Hornbeck, David. (2013). *Mathematics, Music, and the Guitar*. Retrieved from: http://jwilson.coe.uga.edu/EMAT6450/Class%20Projects/Hornbeck/Math,%20Music,%20&%20Guitar.pdf
- Next Generation Science Standards. (2013). Retrieved from: http://www.nextgenscience.org/

Activity:

Introduction: The Helmholtz resonator is both a simple introduction to acoustic resonance and a useful model of many physical systems. These include such diverse examples as engine intake manifolds and the bodies of stringed musical instruments. A pop bottle with a roughly cylindrical body and a short cylindrical neck is a good example since it is readily available and its acoustic response is intuitively familiar. Using several different fluid levels to vary the enclosed air volume, the parameters in the Helmholtz equation can be identified so that the calculated frequencies are within a few percentage points of the measured ones. (French, 2005). *Optional: Students may experiment with a water bottle of their own during the video, if desired.*

Play You Tube Video: Dr. Mark French, *Helmholtz Resonator:* https://youtu.be/s2ocI9AV0KA
"The Helmholtz resonator may be the simplest acoustic device and is used in an approximate form for everything from speaker design to automotive intakes. I show where the idea came from and show the expression for the resonant frequency. I also do a simple demonstration." (French, 2016)

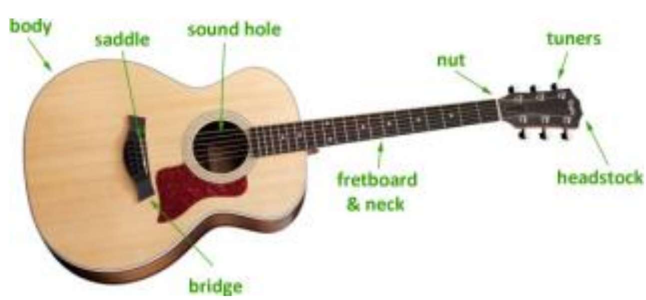
A brass, spherical Helmholtz resonator based on Hermann Helmholtz's original design, circa 1890-1900.

The body of an acoustic instrument behaves very much like a Helmholtz resonator. When pressure is put on the air at the top of the tube, the pressure in the cavity increases and air is shot back out of the tube. This process repeats as the pressure inside and outside of the tube and cavity increase and decrease, and the result is an oscillating sound wave. Helmholtz resonators satisfy the following equation:

$$f = \frac{c}{2\pi}\sqrt{\frac{S}{VL}}$$

where ***c*** is the speed of sound, ***S*** is the surface area of the opening of the tube, ***V*** is the volume of the cavity, and ***L*** is the length of the tube. Because the walls of a guitar/ukulele body are designed to vibrate and are not actually rigid, the calculations will have a certain small percentage of error. However, for the purposes of guitar manufacturing, this error is not a deterrent. (Hornbeck, 2013).

The major parts of a traditional acoustic guitar



Calculate the guitar body's volume and apply the Helmholtz formula to find a guitar's frequency.

Step 1. Measure the diameter of the sound hole and use the radius to calculate the area (as a flat circle).

Step 2. Calculate the surface area of the top of the guitar body. Because the guitar is an irregular shape, approximation methods are needed. Examples of two different methods for finding the surface area of the top are shown below, or you may use another method. The first method divides the guitar body into rectangles. A second and somewhat more accurate method is to consider the guitar's shape as two cylinders and then use the two circles on the top to approximate the surface area.

Rectangle Method

Circle Method

For the circle method, mark the points in the middle of each half of the guitar to indicate the estimated centers of each circle. The strips on the edges of the guitar indicate points on the approximate circumferences of the two circles. After measuring the distances from each center to the corresponding circumference points of that circle, add the numbers together and divide to find the two arithmetic means to get an estimate for the radius of each circle. Calculate the two circle areas, then add them together to find the total surface area of the top of the guitar body.

Step 3. Find the height of the guitar by measuring at several different points on each half of the body, then calculating the arithmetic mean of these to get an average measurement for the height, since the top and bottom of the guitar are not parallel and have a slope to them. Multiply the surface area of the top of the guitar body by the average height of the guitar to determine the volume of the guitar body.

Step 4. The Helmholtz formula requires values for ***c*** (the speed of sound), ***V*** (the volume of the guitar body resonator), ***S*** (the area of the sound hole, or the "tube" into the resonator), and ***L*** (the length of the tube). Since the speed of sound (at sea level) equals approximately ***343 meters per second***, convert all of the measurements into meters. Because the air travels a little farther than the length of the tube, the value of ***L*** must be adjusted to compensate for what is called "end effect," which is a property of the wave travel out of the tube. To do this, multiply the radius ***r*** of the sound hole by ***1.7***.

Step 5. Find the resonant frequency of the guitar by substituting the numerical values into the Helmholtz formula above. The units should cancel out perfectly, giving an answer in hertz. To determine if your answer is reasonable, note that standard tuning of traditional acoustic guitars is 110 Hz (A2 on the 5th string) and on a 4-string soprano, tenor, or concert ukulele is 196 Hz (key of C, low G on the 4th string).

Quiz:

- Include at least 10 quiz questions with answer key. (Questions must be Multiple Choice, and/or Matching).

Reviewing Faculty Cohort Members:

- Include at least two names and schools of reviewing faculty cohort members (refer to email list for faculty cohort member email addresses).