Lesson: Subtraction of Fractions
Math Academy – Day 07
Student Notes

Goals:

- Define subtraction of fractions as separating parts referring to the same whole
- Use equivalent fractions with subtraction of fraction problems involving unlike denominators
- Demonstrate subtraction of fractions with models, drawings, and equations

Prerequisite Knowledge:

- Understand when two fractions are said to be equivalent.
- Be able to find equivalent fractions.

Activities:

1. Whole Class Discussion: Back on day 2, we discussed the physical action associated with subtraction. What does this look like?

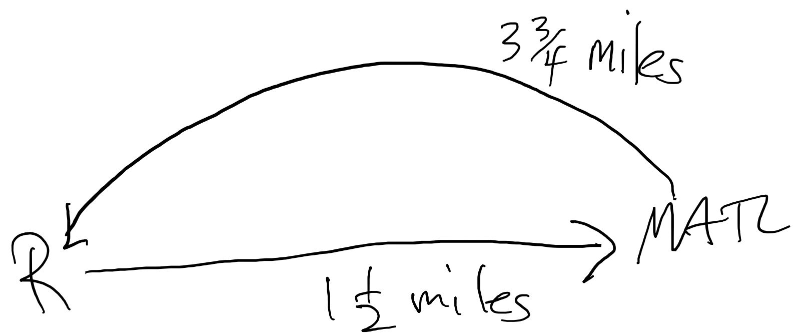
2. Consider the following scenario:

   Tevin drives to MATC each day.
   It is a 1 and ½-mile drive to the downtown campus from his place of residence.
   One day, Tevin took a different route back to his residence which was 3 and 3/4 of a mile.
   How much longer is the route back than the route to MATC?

   a. Draw a picture of this scenario. Be sure to show and label each part of his route.

b. Discuss with your partner the result. Report the result as a fraction (either proper or improper).

c. Matt drew and arrived at the following. Discuss with your partner whether you agree or not. If you do not agree with Matt's conclusion, where did he go wrong and why?



$$3 - 1 = 2$$

$$\frac{3}{4} - \frac{1}{2} = \frac{2}{2}$$

$$2 \cdot \frac{2}{2} = \frac{4}{2} = 2$$

d. Whole Class Discussion: How do we solve this problem? Is Matt correct?

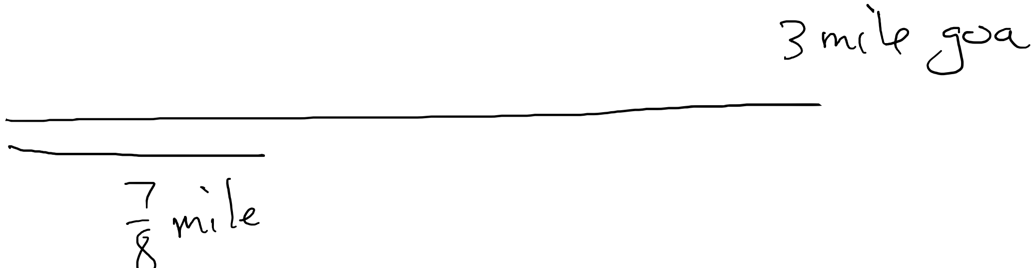
3. Consider the following scenario:

Michael plans on running 3-miles today.
After running 7/8 of a mile, he takes a break.
How much further does Michael plan on running to reach his goal for the day?

   a. Draw a picture of this scenario. Be sure to show and label each part of his route.

   b. Discuss with your partner the result. Report the result as a fraction (either proper or improper).

   c. Missy drew and arrived at the following. Discuss with your partner whether you agree or not. If you do not agree with Missy's conclusion, where did she go wrong and why?



$$3 - \frac{7}{8} = \frac{3}{1} - \frac{7}{8} = \frac{4}{7} \text{ of a mile}$$

   d. Whole Class Discussion: How do we solve this problem?

4. Working with a partner, write your names on the piece of paper given to you by your instructor. You are tasked with creating a story problem that involves subtraction of fractions. Try to be creative and create new scenarios other than the ones provided before. (Note: These will be shared with the class.)

5. Whole Class Discussion: If we are given two fractions, how would we:
    a. Add them? What does it mean to ‘add them together’?

    b. Subtract them? What does it mean to ‘subtract them’?

    c. What are the similarities and differences between addition and subtraction of fractions?

    d. Why don’t we add/subtract denominators when we write the math out?

Goals:

- Define subtraction of fractions as separating parts referring to the same whole
- Use equivalent fractions with subtraction of fraction problems involving unlike denominators
- Demonstrate subtraction of fractions with models, drawings, and equations

Prerequisite Knowledge:

- Understand when two fractions are said to be equivalent.
- Be able to find equivalent fractions.

Lesson Materials:

- Student Notes for Day 07
- Extra fraction strips if students want to use them for thinking through a problem.

Lesson Breakdown:

Activity	Size of Group	Time in Activity Total Time: 55 minutes
Physical action associated with subtraction	Whole Class	3 minutes
Tevin problem	Partners, part (d) as whole class	12 minutes
Michael problem	Partners, part (d) as whole class	12 minutes
Creation of story problem	Partners	20 minutes
Whole Class Discussion	Whole Class	8 minutes

Activities:

1. Whole Class Discussion: Back on day 2, we discussed the physical action associated with subtraction. What does this look like?

   Subtraction is taking away (aligning two amounts, then where they match up, we take those blocks away). Or taking from (given an amount, I take an amount from it).

2. Consider the following scenario:

   Tevin drives to MATC each day.
   It is a 1 and ½-mile drive to the downtown campus from his place of residence.
   One day, Tevin took a different route back to his residence which was 3 and 3/4 of a mile.
   How much longer is the route back than the route to MATC?

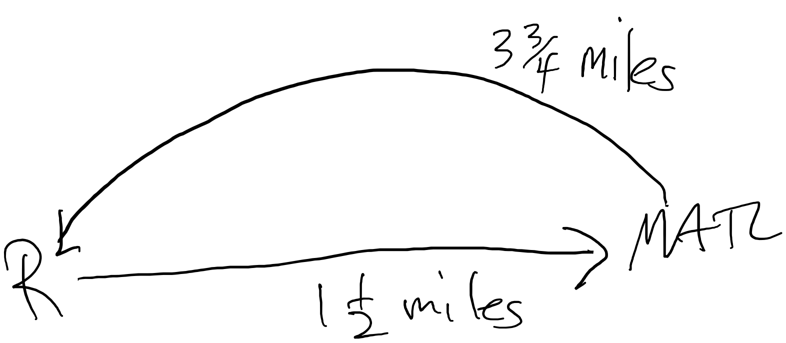
   e. Draw a picture of this scenario. Be sure to show and label each part of his route.

      Here we expect students to start drawing lines (because of how Google maps and other GPS devices work) but drawing lines is NOT necessary. Allow students to be wrong in this drawing, if they are wrong, probe their thinking. Ask them questions that engage their logic, but do not directly tell the student that they are "wrong" (even if they ask). Ask about money, if necessary.

   f. Discuss with your partner the result. Report the result as a fraction (either proper or improper).

      Result: 9/4. If students have 2 and ¼ listed down on their paper, ask them if that is a proper or improper fraction (note: a mixed number contains a proper [or improper] fraction but is not either of them). Ask them to convert their mixed number to an improper fraction if they have a mixed number. Again, don't tell them how to do the conversion. Let them sit and think about it! Tell them (only if they need the help) to refer back to problem 4 from the first hour to help them.

   g. Matt drew and arrived at the following. Discuss with your partner whether you agree or not. If you do not agree with Matt's conclusion, where did he go wrong and why?



$$3 - 1 = 2$$

$$\frac{3}{4} - \frac{1}{2} = \frac{2}{2}$$

$$2 \cdot \frac{2}{2} = \frac{4}{2} = 2$$

   h. Whole Class Discussion: How do we solve this problem? Is Matt correct?

      All of this should be coming from the students. If not, probe for these answers. Do not tell them the answer. No, Matt is not correct. His picture is off because it is hard to compare the two lines that he drew. One line is straight while the other one is curved. So, his drawing does not adequately show justification of his math. Again here, if students used the lining up then taking away method, then it would be easier to show that you have to have similar size regions when you take away. In the base-ten blocks, we did not align a ‘hundred’ with a ‘ten’ and then take them both away because there was “one of them”. We broke the hundred block up into tens and then took one of those tens away. Same holds true here. If we are taking ½ away from ¾, then we need to have similar sized objects (break the ¾ into ½ and ¼) and then take the ½ away.

3. Consider the following scenario:

Michael plans on running 3-miles today.
After running 7/8 of a mile, he takes a break.
How much further does Michael plan on running to reach his goal for the day?

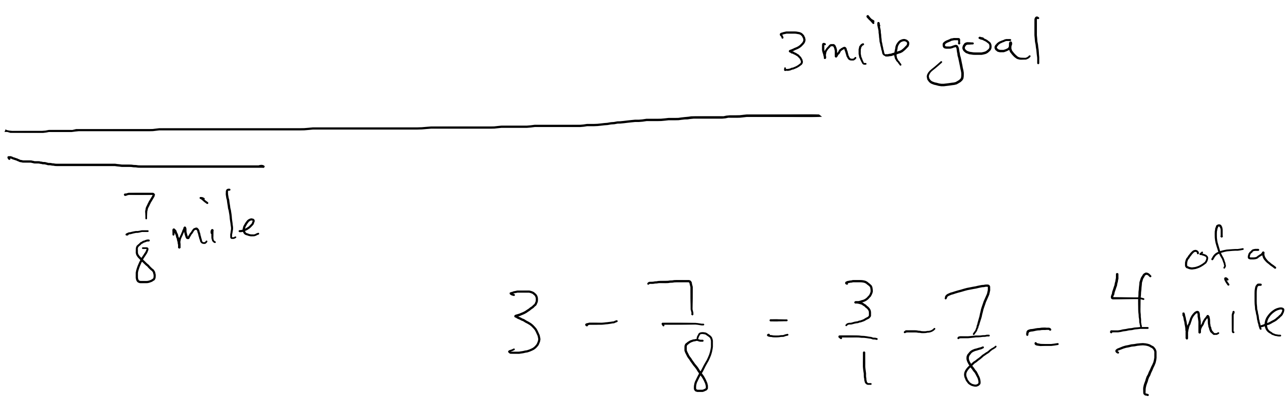
e. Draw a picture of this scenario. Be sure to show and label each part of his route.

This problem also deals with taking a fraction away from a natural number. We again, expect students to draw lines, but it is not necessary. Students may realize that aligning lines makes it difficult to measure, if they are not appropriately marked (like the number line! And there’s a reason for that! Right?). We want students to draw a drawing that would be helpful to solve this situation. If students are not amending their drawings to make sense of the problem, then push them gently into that line of thinking. Say something like: “The reason we ask students to draw is to make sense of the math and the situation in terms of being able to see or visualize your their mind’s eye sees. I want to see how your thinking about math, can you try to draw this scenario so that I can make sense of the math behind your drawing?”

f. Discuss with your partner the result. Report the result as a fraction (either proper or improper).

Result: 17/8 (Again, not 2 and 1/8 which is a mixed number).

g. Missy drew and arrived at the following. Discuss with your partner whether you agree or not. If you do not agree with Missy’s conclusion, where did she go wrong and why?



h. Whole Class Discussion: How do we solve this problem?

There are many issues with what Missy drew. Her drawing didn’t help in the fact that the lines didn’t help her gauge how much she was taking away. Also, in her work, she subtract backwards? 3 – 7 = 4? Try this with students to start to get them to think of signed numbers, ask students to take out 3 objects, then take away seven objects. Does that leave them with four objects?

Have the students discuss, what would the math look like in this situation? What would we write down in order to compute 3 – 7/8. Students may see the need for writing 3 as 3/1, but hopefully they will not need that “step”. Hopefully, they would be able to say that 3 is the same thing as 24/8 if I have to cut each whole into eighths in order to do the subtraction. We would like to see them be able to do: 3 – 7/8 -> 24/8 – 7/8 =

17/8. This would show a strong grasp that a fraction represents a number.

4. Working with a partner, write your names on the piece of paper given to you by your instructor. You are tasked with creating a story problem that involves subtraction of fractions. Try to be creative and create new scenarios other than the ones provided before. (Note: These will be shared with the class.)

   Walk around the room and ensure students are using the operation of subtraction. The students can use any types of numbers that they would like as long as one of the numbers is a fraction.

   We suggest that you present these as a class, have them work through the problems one-by-one with their partners or individually and have them post a drawing along with the result.

5. Whole Class Discussion: If we are given two fractions, how would we:
   a. Add them together? What does it mean to 'add them together' (i.e. what are we physically doing when we do addition of fractions)?

      To add numbers together, we physically combine the two quantities, sort by type, and then count. If we have two fractions that we are still combining together, but in order to count, we have to have similar size regions of counting (just like when we counted with base-ten blocks. We sorted the ones, tens, and hundreds, then counted each type – and we use place value to show the location of the amount of each type). Here, with fractions, the denominators tells us the size of each region and the numerator tells the count of regions. So, location of numbers is important (the numerator represents the count, and the denominator represents the cut size or size of region).

   b. Subtract them? What does it mean to 'subtract them'?

      Subtraction we are physically taking something away from something else. In order to take away the right amount, we also need similar size regions so that we take the appropriate amount away. (For example, if I owe one quarter but I only have a dollar, I don't want people to take my one dollar away because it represents 1 item. I want someone to be able to break my dollar down into four quarters, and then take the one quarter away that I owe.) This holds true for all scenarios of taking away. In order to take the right amount away, if we don't have that amount represented in our original amount, then we need to break something to get it. If we refer to the Michael problem, he needed to break his 3 mile run into eighths of a mile in order to count. If he runs 3 miles then that's the same as running 24/8 miles. Representing 3 as 24/8 makes it easy to take away 7/8.

   c. What are the similarities and differences between addition and subtraction of fractions?

      Similarities: both need to have similar size regions. Common denominators are necessary. Also, both involve counting.

      Differences: With addition, we combine two quantities and count the total. With subtraction, we are taking away and counting what is left over.

   d. Why don't we add/subtract denominators when we write the math out?

      Because denominators are used to tell use the size of the regions (or the cuts/folds of the whole). They are not a part of the "counting process". The numerators tell us how much or how many of the sized regions we have or are considering. It is the numerator that either grows when combining or shrinks when taking away.

Name: ________________________________

1. Sally would like to put chocolate chips and walnuts in her banana bread. She adds 2/3-of a cup of chocolate chips and 3/4-of a cup of walnuts to the recipe. What is the total of these two ingredients?

a. Draw a picture of this scenario. (Make sure both portions are shown and labeled)

b. Compute the result. Report the result as a fraction (either proper or improper).

2. Damian bought 2 and ½ pounds of grapes at the farmers market. On the way home, he ate 1 and 2/3 pounds of the grapes. How much of the grapes remains?

a. Draw a picture of this scenario. (Make sure both portions are shown and labeled)

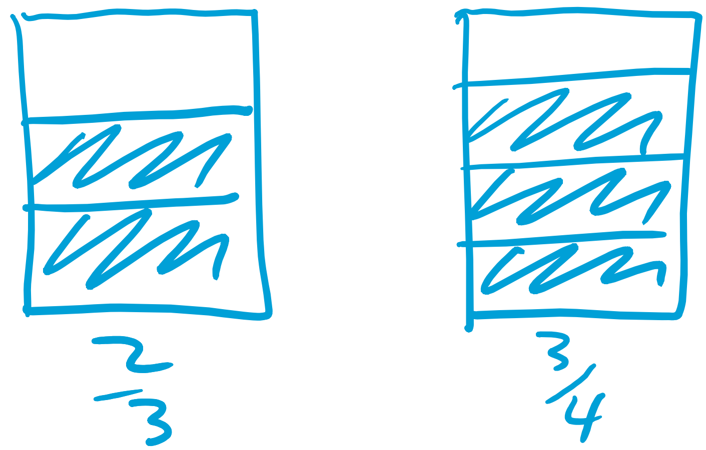
b. Compute the result. If the result is larger than 1, then report the answer as an improper fraction. If not, report the answer as a proper fraction.

MATHPH-700 Math Academy Name: ______________________________

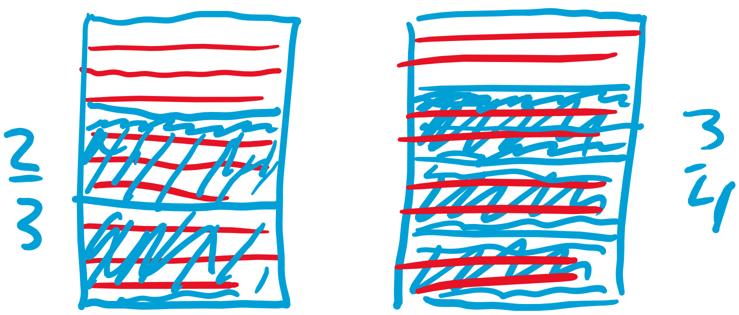
Day 07 Homework Solutions– Addition/Subtraction of Fractions

1. Sally would like to put chocolate chips and walnuts in her banana bread. She adds 2/3-of a cup of chocolate chips and 3/4-of a cup of walnuts to the recipe. What is the total of these two ingredients?

a. Draw a picture of this scenario. (Make sure both portions are shown and labeled)



b. Compute the result. Report the result as a fraction (either proper or improper).



$\frac{8}{12} + \frac{9}{12} = \frac{17}{12}$

We need pieces to be of same size so $\frac{2}{3} = \frac{8}{12}$ and $\frac{3}{4} = \frac{9}{12}$

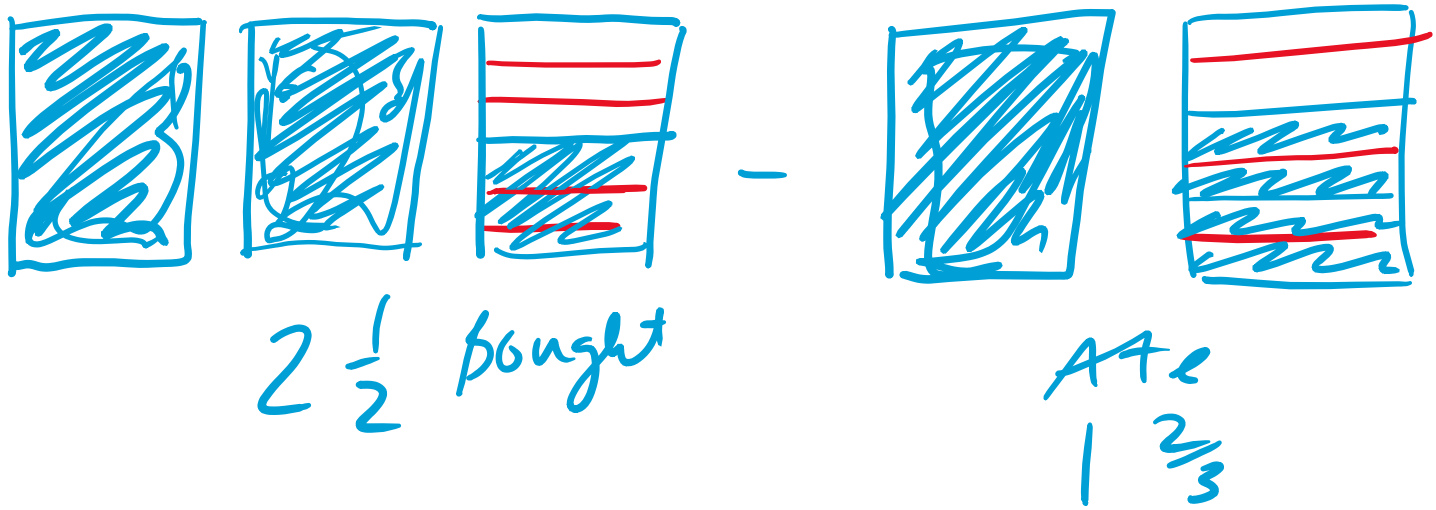
If you look at the drawing you see 17 colored pieces

We needed to cut each $1/3$ into 4 pieces and each $1/4$ into 3 pieces of equal size. (shown in red)

$\frac{17}{2}$ pounds or $1\frac{5}{12}$ lbs

2. Damian bought 2 and ½ pounds of grapes at the farmers market. On the way home, he ate 1 and 2/3 pounds of the grapes. How much of the grapes remains?

a. Draw a picture of this scenario. (Make sure both portions are shown and labeled)



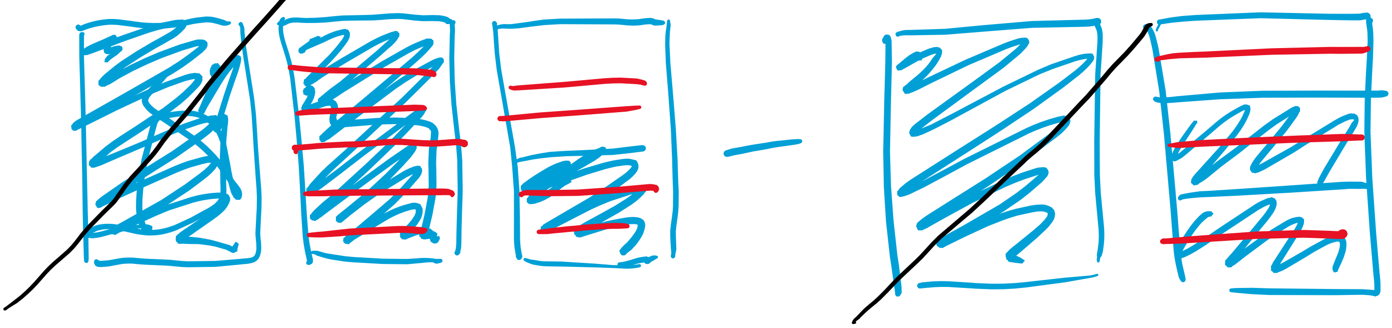
b. Compute the result. If the result is larger than 1, then report the answer as an improper fraction. If not, report the answer as a proper fraction.

$$2\tfrac{1}{2} - 1\tfrac{2}{3} = \frac{5}{2} - \frac{5}{3}$$

$$= \frac{3}{3}\cdot\left(\frac{5}{2}\right) - \frac{2}{2}\left(\frac{5}{3}\right)$$

$$= \frac{15}{6} - \frac{10}{6} = 5/6 \text{ of a pound}$$

need pieces of same size



Remove 1 [ ] from each