

Lesson 8: Percent Slope

PREREQUISITE ASSUMPTIONS

Before beginning this lesson, students should

- ⌘ Know how to convert a fraction to a percent
- ⌘ Understand that right triangles are connected to the language of ‘rise’ and ‘run’

Competencies covered in the lesson and associated homework

Students will be able to:

Unit 4. Percent Problems

A. Perform basic mathematical operations with numbers expressed as percents

- A.1. you convert fractions or decimals into percentages and vice versa
- A.2. you solve percent problems for the missing variable (amount, base, or percentage)
- A.3. you use a scientific calculator to compute the answer to percent problems

B. Use percents in solving word problems

- B.1. you translate a verbally stated application involving percentages into performing an equivalent computation
- B.3. you interpret the computed answer to a word problem

Notes to Self

- One thing I want to do during this lesson
- One thing I want to pay attention in my students’ thinking ...
- One connection or idea I want to remember ...

Suggested Timeline

Duration	Activity (Indicate question number)	Suggested Structure (Indicate group, whole class or individual work)
5 minutes	Sidewalk Definitions	Whole Class
5 minutes	Questions 1 - 4	Groups
7 minutes	Review Questions 1 - 4	Whole Class
5 minutes	Questions 5 - 7	Groups
5 minutes	Review Questions 5 - 7	Whole Class
3 minutes	Question 8	Groups
3 minutes	Review Question 8	Whole Class
3 minutes	Question 9	Groups

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4 minutes	Review Question 9	Whole Class
5 minutes	Making Connections	Whole Class

[Student Handout]

SPECIFIC OBJECTIVES

By the end of this lesson, you will understand that

- % slope, pitch per foot and total fall are related quantities

By the end of this lesson, you will be able to

- Given one item (% slope or pitch per foot or total fall) calculate the other two

Definitions:

- Percent Grade (a.k.a. % Slope)
- Pitch per foot
- Total fall

Duration	Activity (Indicate question number)	Suggested Structure (Indicate group, whole class or individual work)
5 minutes	Definitions	Whole Class
<p>Notes: Start the lesson asking students what they already know. Try to elicit definitions from the class. Below is one possible strategy...</p> <p>Ask the students <i>“has anyone has done sidewalk work?”</i></p> <p>Acknowledge those students and ask them to help with common language used for the work. Draw a slanted line on the board and ask for words that would go along with that for a sidewalk.</p> <p>Possible responses: Slope, Pitch, Grade</p> <p>Ask: <i>“Can anyone help define these for all of us?”</i> and point to the words that need Definitions</p> <p>% Grade: How much slope/incline in the material there is over the entire length of the segment.</p> <p>Be sure to make it clear (if it doesn't come out): A textbook might define it as: % grade = slope as a %. Divide Rise by the run</p>		

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Show an image of a truck-on-wedge road sign and ask: “what do you think it would feel like to drive down that road.”

Pitch per foot: is the level of incline. Change in slope... per unit

Be sure to make it clear (if it doesn't come out): *A textbook definition is: Amount of rise or fall in one foot of run (12")*

Draw out a little triangle on the slanted line on the board to demonstrate the fall.

Total fall: total change in elevation from highest point to lowest point.

Also point out (if it doesn't come out...) make a triangle out of the entire slanted line on the board, and the height is the total fall.

Problem Situation #1 : Investigating Pitch per Foot

You are given a set of plans that have a pitch per foot of $\frac{1}{4}$ " for a section of concrete. Determine the total fall for the following lengths of concrete (aside: the minimum fall for any sidewalk, by code, is $\frac{1}{4}$ " per foot):

Duration	Activity (Indicate question number)	Suggested Structure (Indicate group, whole class or individual work)
5 minutes 7 minutes	Questions 1-4 Review Questions 1 - 4	Groups Whole Class
Notes: Problems 1 – 3 give students a chance to apply their definitions for total fall and pitch per foot and reflect on how they are related. Question #4 asks the students to articulate exactly how they are related.		

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Have students work in groups on problems 1 – 4. As students finish #1 - #3, **ask a couple of groups to put their work on the whiteboards.** Ideally, look for the two different strategies for #1.

The answer from question 1 came straight out of the problem situation since it says that the pitch per foot is $\frac{1}{4}$ " and the sidewalk length for #1 is one foot.

For #2 and #3, there is a possible error that could come up with the construction master calculator.

possible error... if they multiply $\frac{1}{4}$ " by 25', the calculator will convert the feet to inches and give something MUCH too large. They are instead multiplying by 25 sets of 1 foot.

Take Away: Can use the idea of similar triangles/proportions to show why the formula given above is true. Note that the total length must be in # of feet, since the 'pitch per foot' is per FEET.

DRAW the similar triangles on the board (also can be shown as a proportion: $\frac{1}{4}" / 1' = x" / 10'$)

Spend a few minutes talking about the idea of 'similarity' in Geometry and facilitate a conversation about where we have used this idea in construction (in the 3-4-5 lesson for sure!) and explain that it doesn't have to be just triangles, it works for any shape. Connect this concept to 'scale' as in a scale drawing.

Do Calculator Demo: Enter: $\frac{1}{4}$ inch PITCH. Then enter the length (10') followed by RUN. Then, hit 'RISE' and it will give total fall.

(Note that in Lesson 6 when we introduced using the calculator keys for the first time, the instructor drew a triangle and labeled it 'rise' 'run' and 'diag'.)

Ask **"Why does the calculator use 'Rise' and 'Run'?" Does anyone remember where in math class they used that before?"**

Hopefully, someone will relate Rise and Run to slope. If nobody does, draw a coordinate plane with a downward-sloping line and ask students what the steepness of the line is called. And then ask how to calculate it to see if that jogs memory.

On the line, demonstrate the $\frac{1}{4}$ " pitch along a 10 foot length.... Draw a little triangle that is $\frac{1}{4}$ " tall and one foot long, and then a big triangle with 10 feet as the run...

1. For one foot of sidewalk, what is the total fall? Be sure to show your calculations along with your answer.

Answer: $\frac{1}{4}$ "

2. For ten feet of sidewalk, what is the total fall? Be sure to show your calculations along with your answer.

Answer: $2 \frac{1}{2}$ "

Strategy #1: add $\frac{1}{4}$ " 10 times

Strategy #2: multiply. $\frac{1}{4}" \times 10 = 2 \frac{1}{2}"$

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Note... it is $\frac{1}{4}$ ", 10 times. Or, remember that it is $\frac{1}{4}$ " PER FOOT, so, when multiplying by 10 feet, the units of feet cancels

3. For 25 feet of sidewalk, what is the total fall? Be sure to show your calculations along with your answer.

Answer: $6\frac{1}{4}$ "
multiply. $\frac{1}{4}" \times 25 = 6\frac{1}{4}"$

4. Explain or show how 'pitch per foot' connects to 'total fall.'

Answer: Pitch per foot \times Total # of feet = Total fall (rise)

Duration	Activity (Indicate question number)	Suggested Structure (Indicate group, whole class or individual work)
5 minutes 5 minutes	Questions 5 - 7 Review Questions 5 - 7	Groups Whole Class
<p>Notes: Problems 5 - 7 give students a chance to connect pitch per foot with % grade and then to articulate how they are related.</p> <p>Students may struggle doing the % calculation... suggestions to spur discussion: To calculate the %, just DO the Division Questions... how do you convert to a % Some may say they move the decimal 2 places (which direction)? Others that they multiply by 100 to convert it to a %</p> <p>Maybe connect to Lesson 7 " ____% OF ____ IS ____"</p> <p>Take Away: The beauty of seeing this connection is that, if they can see it, then, they know that % slope is always $x" / 1 \text{ foot}$</p> <p>So DEMONSTRATE... 2% is 0.02 ft "pitch per foot" so, just convert 0.02 from ft to inches (use calculator) !!! 0.02 feet is 0.24" or $\frac{1}{4}"$ pitch per foot</p>		

5. Now, using your definition for % slope (Percent Grade), what is the % slope for the 25' length sidewalk? Be sure to show your calculations along with your answer.

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Answer: 2%

According to the definition, it is Rise divided by Run. So $6.25'' / 25' = 0.02083333$

Convert to a % by multiplying by 100: 2%

6. What is the % slope for the 1' length? Be sure to show your calculations along with your answer.

Answer: 2%

According to the definition, it is Rise divided by Run. So $0.25'' / 12'' = 0.02083333$

Convert to a % by multiplying by 100: 2%

7. Explain or show how the % slope connects the pitch per foot?

Answer: They are the same! The % slope is the actual division written as a %

Connecting instead to total fall... total fall divided by total run = slope %

Problem Situation #2 : Forming Up a Sidewalk: Given % Grade

You are a carpenter on a flatwork crew that is tasked with placing a sidewalk. Your crew lead has given you the following information: The total length for the section of sidewalk you need to form up is 50 feet with a required 2% grade.

Duration	Activity (Indicate question number)	Suggested Structure (Indicate group, whole class or individual work)
3 minutes 3 minutes	Question 8 Review Question 8	Groups Whole Class
<p>Notes: This problem situation has only a single question. It is asking students to build on their new knowledge from the first problem situation.</p> <p>Have students do Problem situation 2 and 3 at the same time (since they are both so short and both build on problem situation 1)</p> <p>Strategy for #8: Students can use any strategy. Want it to come out that there is an actual formula To calculate: there is a formula... total length x % grade = total fall. So, $50' \times 0.02 = 1$ foot or 12 inch fall</p> <p>Take Away: A lot of times, a formula is just there to let you choose not to think. (not always). The point is that formulas are based on reality. If you understand the reality of the situation really well, a lot of the time you can figure out the formula.</p>		

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Another example (ask for students to share these): perimeter, area of a rectangle or square... (this is also a prelude to the baluster formula).

8. What is the total fall of the sidewalk for that 50'? Be sure to show your calculations along with your answer.

Answer: 12 inches
 $50' \times 0.02 = 1 \text{ foot or } 12 \text{ inch fall}$

Problem Situation #2 : Forming Up a Sidewalk: Given Total Fall

You are a carpenter on a flatwork crew that is tasked with placing a sidewalk. Your crew lead has given you the following information: The total length for the section of sidewalk you need to form up is 40 feet. The total fall for that section is 24".

Duration	Activity (Indicate question number)	Suggested Structure (Indicate group, whole class or individual work)
3 minutes 4 minutes	Question 9 Review Question 9	Groups Whole Class
<p>Notes: This problem situation has only a single question. It is asking students to build on their new knowledge from the first problem situation.</p> <p>Have students do Problem situation 2 and 3 at the same time (since they are both so short and both build on problem situation 1)</p> <p>Strategy for #9: Students need to remember how to calculate % slope. The method of using total rise and total run is the simplest. They need to be sure to use the unit buttons on their calculators, or convert the number of inches of rise to feet.</p> <p>Again here, students may struggle with how to calculate %. Encourage students to share strategies. Ask ALL groups to put their work on the board for #9. Highlight the different methods students used.</p> <p>Make sure all students have written down HOW to find % (1. DO the division and then 2. Multiply by 100)</p>		

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9. Building code says that if Percent Grade is 5% or greater, then the sidewalk is considered a ramp and needs guardrail. Will this section of sidewalk require a guardrail? Be sure to show your work to justify your answer.

Answer: YES

To calculate % grade given fall and length: $\text{fall} / \text{length} \times 100\%$

So: $24'' = 2'$ so $2' / 40' \times 100\% = 5\%$ grade

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

State the main idea of the lesson

Total Fall, % Slope and Pitch Per Foot are all related through triangles. Knowing one, the other two can be calculated.

Practice/Homework

1. Pitch Per Foot = $\frac{1}{2}''$. Calculate % Slope.
2. % Slope = 5%. Calculate Pitch Per Foot.
3. Pitch Per Foot = $\frac{3}{8}''$. Calculate Total Fall for a 60' run of sidewalk. Calculate % Slope.
4. Total Fall = 12'' and the sidewalk run = 40 feet. Calculate % Slope. Calculate Pitch Per Foot.
5. You have poured a 30' length of sidewalk with an 11'' fall. What is the % slope?
6. The pitch per foot of a driveway is $\frac{3}{4}''$.
 - a. What is the fall for 30' of driveway?
 - b. What is the % slope of this driveway?
 - c. If there is a sidewalk placed next to the driveway, will it require a guardrail?