LEO 104: Photonics Concepts Learning Unit 04, Fundamentals of Light & Lasers 3e, Module 1.4.

Study Guide: Geometrical Optics

1. Distinguish between light rays and light waves.
A light ray is nothing more than an imaginary line directed along the path that light energy follows. It is helpful to think of a light ray as a thin pencil of light, very much like a narrow, well-defined laser beam. In the study of geometrical optics, we find it useful to represent the interaction of light waves with plane and spherical surfaces—such as those found in mirrors and lenses—in terms of the bending of light rays. Geometrically, then, a ray is a line perpendicular to a series of successive wave fronts. The direction of the ray specifies the direction of energy flow in the wave. Study Figure 4-2.

2. State the law of reflection.
From a plane surface: When light reflects from a plane surface, as shown in Figure 4-5a, the angle (B) formed by the reflected ray and the normal (line perpendicular to the surface) at the point of incidence is always equal to the angle (A) formed by the incident ray and the same normal. Note carefully that the incident ray, reflected ray, and normal always lie in the same plane.
From a curved surface: With spherical mirrors, reflection of light occurs along a curved surface. The law of reflection holds, since at each point on the curved surface one can draw a surface tangent (a plane surface) and erect a normal to a point P on the surface where the light is incident, as shown in Figure 4-6. One then applies the law of reflection at point P just as was illustrated in Figure 4-5, with the incident and reflected rays making the same angles (A and B) with the normal to the surface at P.

3. State Snell's law of refraction. The bending of light rays at an interface between two optical media is called refraction. Figure 4-8: Note that in the first case (lower-to-higher), the light ray is always bent toward the normal. In the second case (higher-to-lower), the light ray is always bent away from the normal. It is helpful to memorize these effects. Law of refraction relates the sines of the angles of incidence and refraction at an interface between two optical media to the indexes of refraction of the two media. Enables us to calculate the direction of the refracted ray if we know the refractive indexes of the two media and the direction of the incident ray. Equation: $n_i \sin i = n_r \sin r$ where i is the incident angle and r is the refracting angle.

4. Define index of refraction and give typical values for glass, water, and air. The two transparent optical media that form an interface are distinguished from one another by a constant called the index of refraction, generally labeled with the symbol n. The index of refraction for any transparent optical medium is defined as the ratio of the speed of light in a vacuum to the speed of light in the medium, as given in the equation: $\mathbf{n = c/v}$
Glass: crown = 1.52/flint = 1.66, Water: 1.33, Air: 1.0003 (commonly accepted as 1.00 as in a vacuum). Study Figure 4-8 and Example 2.

5. Explain the critical angle of incidence for the interface between two optical media. When light travels from a medium of higher index to one of lower index, we encounter some interesting results. Study Figure 4-10. Ray 3 is incident at the critical angle ic, large enough to cause the refracted ray bending away from the normal to bend by 90°, at angle r_c, thereby traveling along the interface between the two media. Study Figure 4-10 and examples 3 & 4 and equation 4-3.

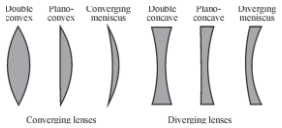
6. Describe the process of total internal reflection. Figure 4: Ray 4 is incident on the interface at an angle greater than the critical angle and is totally reflected into the same medium from which it came. Ray 4 obeys the law of reflection so that its angle of reflection r_4 is exactly equal to its angle of incidence i_4. Study Figure 4-10 and examples 3 & 4.

7. Describe how total internal reflection can be used to trap light in fibers for use in telecommunication. The phenomenon of total internal reflection explains the light propagation in an optical fiber by "trapping" the light in the fiber through successive internal reflections along the fiber. Study Figure 4-10 and examples 3 & 4.

8. Describe dispersion of white light. The refractive index is slightly wavelength dependent so values of *n* change a little as λ changes. For example, the index of refraction for flint glass is about 1% higher for blue light (around 450 nm) than for red light (around 650 nm). The variation of refractive index *n* with wavelength λ is called *dispersion*. Figure 4-12b shows the separation of the individual colors blended together in white light—400 nm to 700 nm—after passing through a prism. Note that n_λ decreases from shorter to longer wavelengths, thus causing the red light to be less deviated than the blue light as it passes through a prism. Dispersion accounts for the colors seen in a rainbow, where the individual raindrops serve as the refracting "prism."

9. Describe the relationship between collimated light and the focal points of convex and concave mirrors. In this basic introduction of geometrical optics, we shall deal only with thin lenses. The *focal points* of lenses are defined in terms of the effect that lenses have on incident parallel light rays (collimated) and plane wave fronts. A lens is made up of a transparent refracting medium, generally of some type of glass, with spherically shaped surfaces on the front and back. A ray incident on the lens bends (according to Snell's law) at the front surface, propagates through the lens, and bends again at the rear surface (according to Snell's law). C*onverging* or *positive* lenses cause parallel rays passing through them to bend toward one another and give rise to *positive* focal lengths. D*iverging* or *negative* lenses cause parallel rays passing through them to spread as they leave the lens and thereby give rise to *negative* focal lengths.

10. Describe the shapes of three typical converging (positive) thin lenses and three typical diverging (negative) thin lenses. The first three lenses are *thicker in the middle than at the edges* and are described as *converging* or *positive* lenses. The last three lenses are *thinner in the middle than at the edges* and are described as *diverging* or *negative* lenses.



11. Describe the relationship between collimated light and the focal points of a thin lens.
Study Figures 4-24a & b. For thin lenses, there are *two* focal points, symmetrically located on each side of the lens, since light can approach from either side of the lens. The sketches in Figure 4-25 indicate the role that the two focal points play, for positive lenses (Figure 4-25a) and negative lenses (Figure 4-25b). Study these figures carefully.

12. Describe the f-number and numerical aperture for a lens and explain how they control image brightness. The transverse dimension (size) of a lens determines its light-gathering power and, consequently, the brightness of the image it forms. The *f-number*, also referred to as the *relative aperture* and the *f/stop*, is defined simply as the ratio of the focal length *f* of the lens, to its transverse diameter *D*, as given in Equation 4-8. Table 4-2 lists the usual choices of *f*/stops (*f*-numbers) and gives the irradiance *E*0 as the value for an *f*/stop of 1 and shows how the image irradiance decreases as the adjustable aperture size behind the camera lens is made smaller so that *D* decreases.
The *numerical aperture* is another important design parameter for a lens, related again directly to how much light the lens gathers. If the focal length of a design lens increases and its diameter decreases, the solid angle (cone) of useful light rays from object to image for such a lens decreases. The definition of numerical aperture (*N.A.*) is given in Equation 4-9 where n is the index of refraction and α is the half angle.
We can *increase* the light-gathering power of a lens and the brightness of the image formed by a lens by *decreasing* the *f*-number of the lens (increasing lens diameter) or by *increasing* the numerical aperture of the lens (increasing an appropriate refraction index and thus making possible a larger acceptance angle).

13. State the ray-tracing rules to locate the images formed by plane and spherical mirrors.
Study Figures 4-15, 4-16, 4-18

To employ the method of ray tracing, we agree on the following:
- Light from an object will be incident on a mirror surface initially from the left.
- The *axis of symmetry* normal to the mirror surface is defined as the *optical axis.*
- The point where the optical axis meets the mirror surface is called the *vertex*, generally denoted by the letter *V*.

For a concave mirror:
- An incident ray from object point *P*, initially *parallel to the axis*, such as ray 1, reflects from the mirror and *passes through the focal point F* (labeled ray 1').
- An incident ray from *P* passing through the focal point *F* on its way to the mirror, such as ray 2, reflects from the mirror *as a ray parallel to the axis* (labeled ray 2').
- An incident ray from *P* passing through the center of curvature *C* on its way to the mirror, such as ray 3, *reflects back along itself* (labeled ray 3').
- Reflected rays 1', 2', and 3' converge after reflection to locate point *P*' on the image. The complete image, arrow *OP*', is a *real* image that can be formed on a screen located there. It is seen as a real image by an eye located where shown in Figure 4-18a.

For a convex mirror:
- An incident ray from object point *P*, initially parallel to the axis, such as ray 1, reflects from the mirror *as if it came originally from the focal point F behind* the mirror (labeled ray 1').
- An incident ray from *P*, such as ray 2, which is headed initially on its way to the mirror in a direction toward the focal point *F* behind the mirror, reflects from the mirror in a direction parallel to the optical axis (labeled ray 2').
- An incident ray from *P*, such as ray 3, which is headed initially on its way to the mirror in a direction toward the center of curvature *C* behind the mirror, reflects back along itself (labeled ray 3').
- Rays 1', 2', and 3' diverge (spread apart) after reflection. A person looking toward the mirror from the left (see eye) intercepts the diverging rays and sees them as if they appear to come from their common intersection point *P*', behind the mirror. The complete image (arrow *IP*') is *virtual* since it cannot be formed on a screen placed there.

Sign convention for spherical mirrors:
- Object and image distances *p* and *q* are both *positive* when located to the *left* of the vertex and both *negative* when located to the *right*.
- The radius of curvature *r* is *positive* when the center of curvature *C* is to the *left* of the vertex (concave mirror surface) and *negative* when *C* is to the *right* (convex mirror surface).
- Vertical dimensions are positive above the optical axis and negative below.
- And since the focal length *f* is equal to one-half of the radius of curvature *r*, i.e., $f = 2r$, as shown earlier in connection with Figures 4-16 and 4-17, the focal length *f* is *positive for concave mirrors* and *negative for convex mirrors*.

14. State the ray-tracing rules to locate the images formed by lenses.
Study Figures 4-22, 4-24, 4-25, 4-27, 4-28, & 4-29

Sign convention (rules) for lenses:
- We draw the "optical system" such that light travels initially from left to right toward the first lens.
- Object distance *p* is *positive* for *real* objects located to the *left* of the lens and *negative* for *virtual* objects located to the *right* of the lens.

- Image distance *q* is *positive* for *real* images formed to the *right* of the lens and *negative* for *virtual* images formed to the *left* of the lens.
- The focal length *f* is *positive* for a *converging* lens, *negative* for a *diverging* lens.
- The radius of curvature *r* is *positive* for a *convex* surface, *negative* for a *concave* surface. (Note that this is the *reverse* of the sign convention adopted for mirrors.)
- Transverse distances (*ho* and *hi*) are *positive above* the optical axis, *negative below*.

15. State the summary of ray optics for mirrors.

Summary of ray optics for PLANE mirrors:

- Each ray of light incident on a mirror from a point on an object obeys the law of reflection at the plane mirror surface.
- A point source of light in front of a mirror forms a virtual image as far behind the mirror as the point source is in front.
- Plane mirrors form virtual images that are the same size as the original objects. Hence, the magnification is unity.

16. Identify the four "special" prisms listed in reading. Figure 4-13: Image manipulation with refracting prisms: Right Angle, Dove, Penta & Porro.

17. The speed of light in a transparent semiconductor material is measured to be 7.37 x 10^7 m/s. What is the index of refraction of this material? What is a good guess for the identity of this material?

n = c/v; n = 3.0 x 10^8 m/s ÷ 7.3 x 10^7 m/s; **n = 4.07, Germanium is 4.1** from Table 4-1.

18. A laser beam is incident on a quartz prism of index n = 1.46 at an angle of incidence of 60°. What are the angles of reflection and refraction at the air-quartz interface?

Angle of reflection = angle of incidence ∴ the **angle of reflection is 60°**.

Angle of refraction: Snell's Law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$ ∴ 1 x sin 60° = 1.46 x sin θ_2 ∴ sin θ_2 = sin60° / 1.46 = **36.4°**.

19. An isosceles prism with equal base angles of 75° produces an angle of minimum deviation of 30° when a laser beam is passed through it. What is the index of refraction of this material? What might the material be?

75° = 75° = 150°, 180° – 150° = 30° ∴

A = 30°

δ = 30°

$$n = sin(\frac{(A + \delta m)}{2})/\sin(\frac{A}{2})$$

30 + 30 = 60, 60/2 = 30, sin30 = .5

30/2 = 15, sin15 = .258819

.5 / .258819 = 1.93 therefore **n = 1.93**

Material: zircon.

20. Based on the dispersion curve for light flint glass given in Figure 4-12, estimate to 2 decimal places the value of n_λ at λ = 400 nm and λ = 650 nm.

400 nm ~ **1.60**

650 nm ~ **1.57**

Helpful TERMS:

Virtual Image
Radius of curvature
Image distance
Real image
Pinhole camera
Specular
Diffuse
Focal point