Wheatstone Bridge Derivation Activity
Instructor Guide

Notes to Instructor

The Wheatstone bridge derivation activity is an activity in the Micro Pressure Sensors & Wheatstone Bridge Learning Module. This activity should be assigned after the Wheatstone Bridge Overview PK. Below are the units in the Micro Pressure Sensor & The Wheatstone Bridge Learning Module:

- Knowledge Probe (KP) pretest
- Wheatstone Bridge Overview PK
- Modeling a Micro Pressure Sensor Activity
- **Wheatstone Bridge Derivation Activity**
- Wheatstone Bridge Final Assessment

Description and Estimated Time to Complete

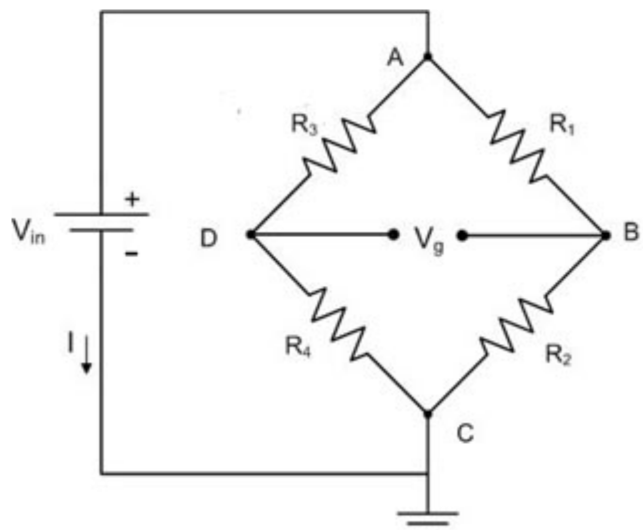
This activity will involve using your knowledge of the Wheatstone bridge to derive the relationship between V_{in} (Input Voltage) and V_g (Gauge Voltage) for a given circuit. You will also design a Wheatstone bridge layout to get the maximum effect on the gauge voltage, V_g.

It is recommended that you review the unit Wheatstone Bridge Overview if you haven't already. It will enhance your understanding of the Wheatstone bridge circuitry as you complete this activity.

Estimated Time to Complete
Allow approximately two hours to complete this activity.

Introduction

The Wheatstone bridge is one of the most sensitive and precise methods of measuring small changes in resistance. This is possible through its use of transducers (devices which change one form of energy into another, such as mechanical to electrical or electrical to mechanical). The Wheatstone bridge incorporates one or more electrical transducers that change resistance as a result of an environmental change or input (e.g. temperature, pressure, stress). This change is sensed by the circuitry of the Wheatstone bridge which provides a useable electrical output (voltage) representative of the input. The Wheatstone bridge is widely used today in macro-sized and micro-sized sensors.



Basic Wheatstone Bridge Configuration

Activity Objectives

Objectives

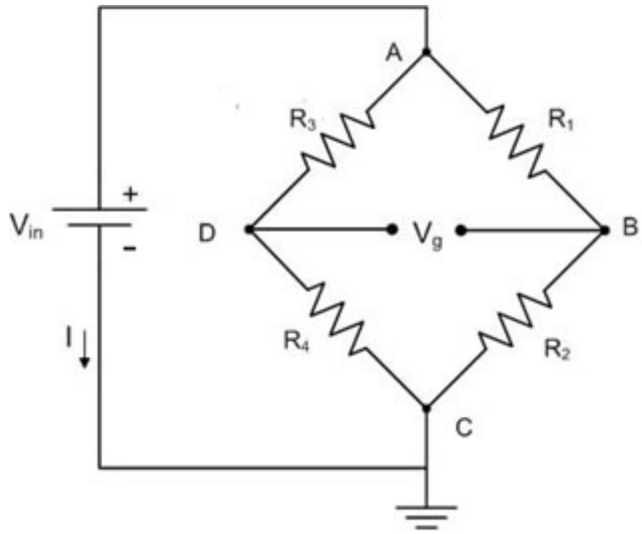
- Derive the mathematical relationship between output voltage (Vg, also referred to as gauge voltage) and resistance in a Wheatstone bridge circuit.
- Design the orientation of the resistors on a pressure sensor membrane to maximize the effect on the gauge voltage.

<u>Procedure I</u>: Derivation Activity

1. Using basic laws of electronics (e.g. Ohm's and Kirchhoff's), derive the mathematical relationship between input voltage, $V_{in,}$ vs. gauge voltage, V_g in terms of input voltage, V_{in} and resistance for the circuit given below.

   $$V_g=V_{in}\,\{f(R_1,R_2,R_3,R_4)\}$$

   Hint: **Starting point would be $V_g = V_D - V_B$.**



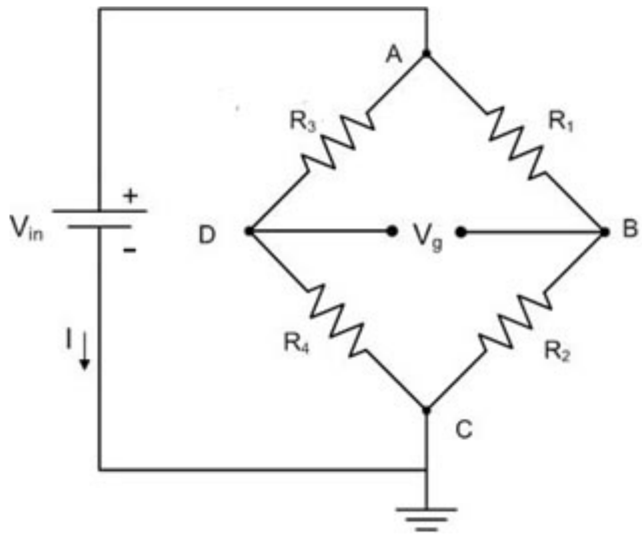
2. Using your results from 1), above, show that the Wheatstone bridge is balanced when $R_3/R_4 = R_1/R_2$.
   a. What values of R_1, R_2, R_3, and R_4 would yield a balanced bridge circuit?
   b. Verify your mathematical derivation using these values of resistors.
   c. Did your derivation yield a $V_g = 0$ V? If not, then find and correct the problem and repeat 2b.
3. Given a Wheatstone bridge with the following V_{in} and R values, what is V_g? ________
   - $V_{in} = 10$ volts
   - $R_1 = 75\ \Omega$
   - $R_2 = 100\ \Omega$
   - $R_3 = 100\ \Omega$
   - $R_4 = 75\ \Omega$
4. If R_2 and R_3 are both variable resistors (transducer elements) and continue to increase above 100 Ω, what effect would it have on V_g?

Answers to Procedure I Questions:

1. Using basic laws of electronics (e.g. Ohm's and Kirchhoff's), derive the mathematical relationship between input voltage, $V_{in,}$ vs. gauge voltage, V_g in terms of input voltage, V_{in} and resistance for the circuit given below.

   $$V_g = V_{in}\{f(R_1, R_2, R_3, R_4)\}$$

   Hint: **Starting point would be $V_g = V_D - V_B$.**



$$V_g = V_D - V_B$$

$$V_D = V_{in}\frac{R_4}{R_3 + R_4}$$

$$V_B = V_{in}\frac{R_2}{R_1 + R_2}$$

$$V_g = V_D - V_B = V_{in}\frac{R_4}{R_3 + R_4} - V_{in}\frac{R_2}{R_1 + R_2}$$

$$V_g = V_{in}\left(\frac{R_4}{R_3 + R_4} - \frac{R_2}{R_1 + R_2}\right)$$

2. Using your results from 1), above, show that the Wheatstone bridge is balanced when $R_3/R_4 = R_1/R_2$.
    a. What values of R_1, R_2, R_3, and R_4 would yield a balanced bridge circuit?

       Answer: Answers will vary, but the ratios should be correct. For example, a correct answer would be $R_1 = 50\ \Omega$, $R_2 = 100\ \Omega$, $R_3 = 5\ \Omega$, $R_4 = 10\ \Omega$.
    b. Verify your mathematical derivation using these values of resistors.
    c. Did your derivation yield a $V_g = 0$ V? If not, then find and correct the problem and repeat 2b. ***Participants should be able to use this step to verify that their derivation, as well as their calculations are correct.***

3. Given a Wheatstone bridge with the following V_{in} and R values, what is V_g? _-1.43 volts
    - $V_{in} = 10$ volts
    - $R_1 = 75\ \Omega$
    - $R_2 = 100\ \Omega$
    - $R_3 = 100\ \Omega$
    - $R_4 = 75\ \Omega$

4. If R_2 and R_3 are both variable resistors (transducer elements) and continue to increase above 100 Ω, what effect would it have on V_g?

   Answer: Vg would become more negative

Procedure II: Design a Wheatstone Bridge for a Micro Pressure Sensor

In the case of a pressure sensor, all resistances are designed with the same geometries, therefore, all the resistors have the same resistance values and the circuit is initially balanced and V_g (gauge voltage) is zero.

Variable resisters in the bridge change due to stress on the resistor's material. This is due to the fact that resistance is a function of the resistor's material (resistivity or ρ), length (L) and cross sectional area (A):

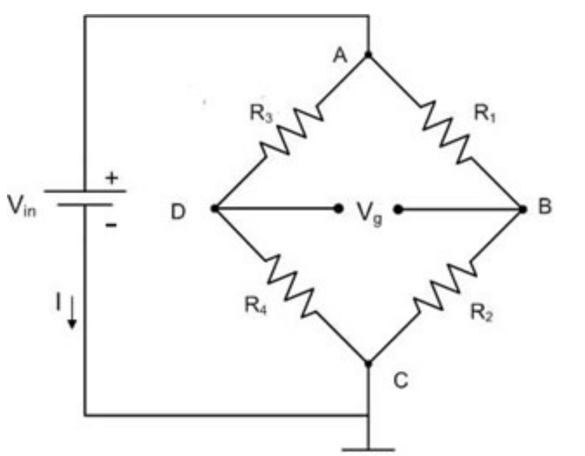
$$R = \rho \, (L/A)$$

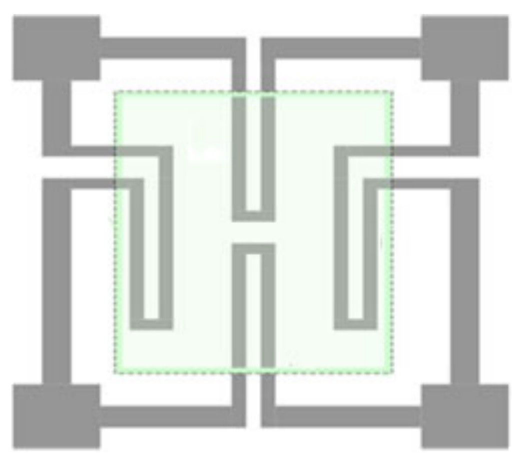
When metal is stressed, it stretches (for instance, a gold resistor on a pressure sensor membrane). The length, L, increases, which results in an increase in resistance, R. The cross-sectional area, A, decreases; however, ρ does not change as it is a property of the material.

In a micro pressure sensor, the Wheatstone bridge can be fabricated so that two of the four resistors are effectively stressed when the membrane is stretched. The stress on the other two resistors is ineffective or negligible. Such fabrication will result in a response from the Wheatstone bridge that is sensitive to small changes in pressure. All four resistors on fabricated on the membrane. Why? Because there is also a temperature dependence of resistance and you want all of the resistors exposed to the exact same temperature environment. All four resistors should be of the same length, width and thickness so they are balanced under no stress. Most pressure sensor systems have an offset circuit to make up for slight variations in the fabrication process.

Questions

1. For maximum effect on the gauge voltage, $V_{g,}$ which two resistors in the schematic (left) could be variable and which two resistors could be fixed?
2. Using the image to on the right, how would you orient the resistors (narrow gray lines) on a membrane (green in the right image)?
    a. On the schematic, indicate the variable resistors with an arrow.
    b. On the pressure sensor diagram, label the four resistors on the diaphragm and the electrical nodes (A, B, C, and D), indicating V_{in} and V_g connections.





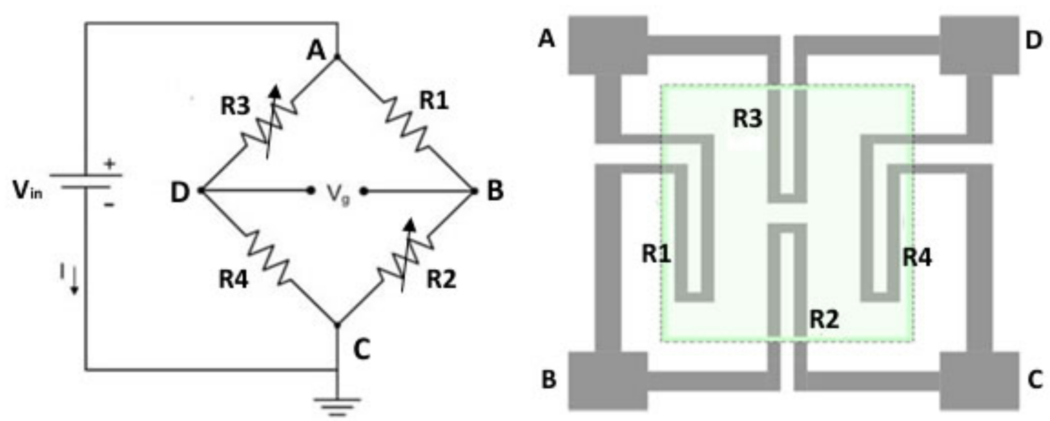
***Answers to Procedure II Questions*:**

1. For maximum effect on the gauge voltage, V_g, which two resistors in the schematic (left) could be variable and which two resistors could be fixed?
   Answer: There are two possible combinations:
   a. Variable resistors - R_2 and R_3, fixed resistors – R_1 and R_4.
   b. Variable resistors – R_1 and R_4, fixed resistors – R_2 and R_3.

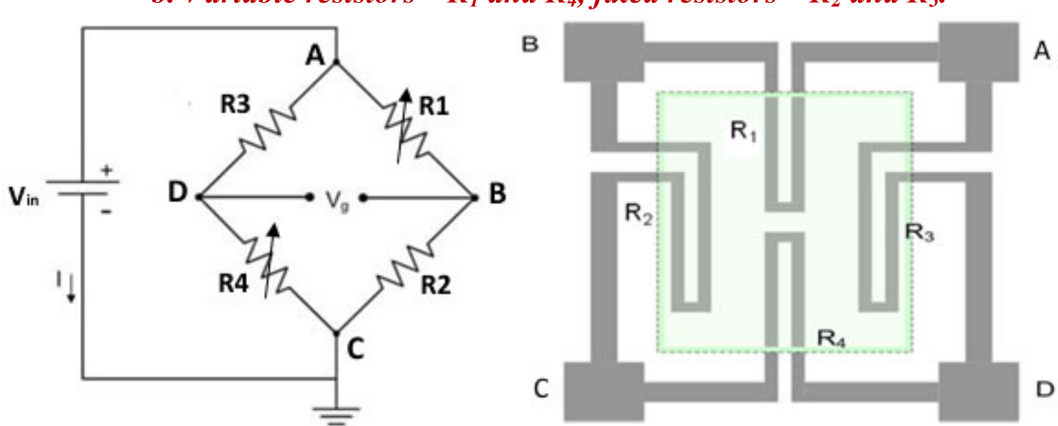
2. Using the image to on the right, how would you orient the resistors (narrow gray lines) on a membrane (green in the right image)?
   a. On the schematic, indicate the variable resistors with an arrow.
   b. On the pressure sensor diagram, label the four resistors on the diaphragm and the electrical nodes (A, B, C, and D), indicating V_{in} and V_g connections.

Answer:
In the event that the membrane is stressed, you want to orient two diagonally opposing resistors to experience a minimum stress and the other two opposing resistors to be oriented for maximum stress.

a. Variable resistors - R_2 and R_3, fixed resistors – R_1 and R_4.

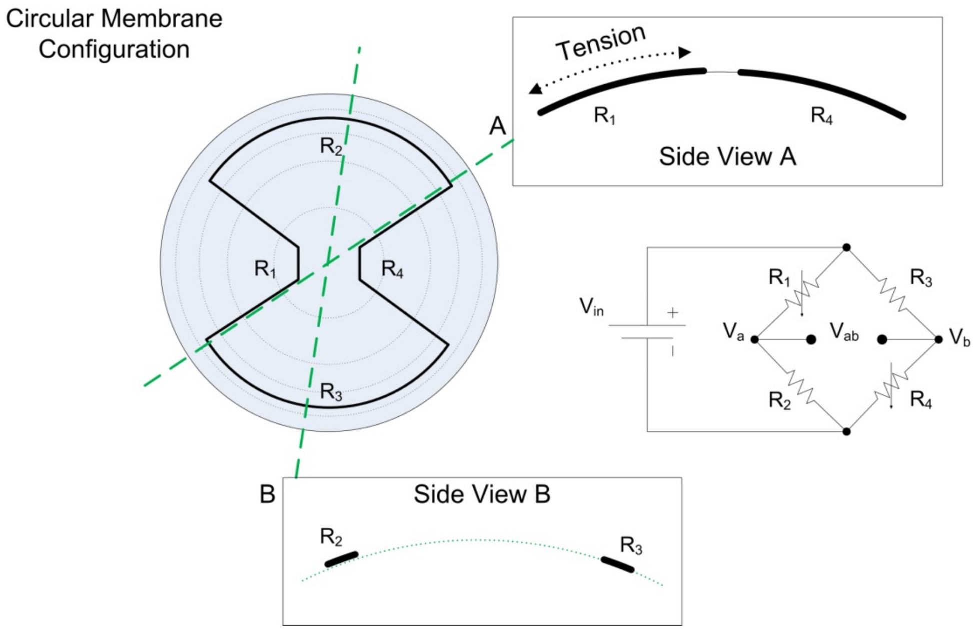


b. Variable resistors – R_1 and R_4, fixed resistors – R_2 and R_3.



Post-Activity Questions

Here is an example of a pressure sensor Wheatstone bridge in a circular configuration. This is the circuit you will build if you complete the "Pressure Sensor Model" Activity. This bridge is created by stretching a balloon membrane across an open can and applying a graphite circuit to the membrane. Note, resistors R_2 and R_3 are configured parallel to the edge of the can, and hence, will not stretch as the membrane expands. R_1 and R_4 resistors are configured over the open part of the membrane or can, parallel to the radius, and will be subject to the highest tension (stretching) and experiencing the greatest piezoresistive effect.



1. Based on the schematic given above, write the equation for V_{ab} in terms of resistance and V_{in}.

2. Sketch a graph that shows the relationship between V_{ab} and Variable Resistance (R_v).

3. How would the graph change if R_2 and R_3 were variable and R_1 and R_4 were fixed? (Justify your answer with formulas and/or a graph.)

Answers to Post-Activity Questions:

1. Based on the schematic given above, write the equation for V_{ab} in terms of resistance and V_{in}.

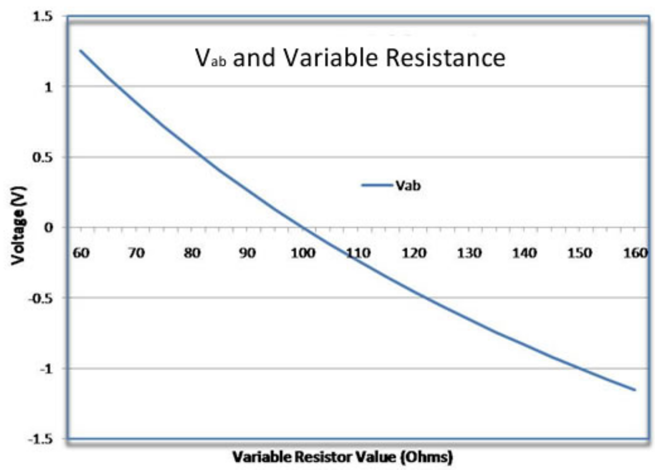
$$V_{ab} = V_a - V_b$$

$$V_a = V_{in} \frac{R_2}{R_1 + R_2}$$

$$V_b = V_{in} \frac{R_4}{R_3 + R_4}$$

$$V_{ab} = V_a - V_b = V_{in} \left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} \right)$$

2. Sketch a graph that shows the relationship between V_{ab} and Variable Resistance (R_v).



3. How would the graph change if R_2 and R_3 were variable and R_1 and R_4 were fixed? (Justify your answer with formulas and/or a graph.)

$$V_{ab} = V_a - V_b = V_{in} \left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} \right)$$

Since R_2 is now variable and R_4 fixed, R_2 will increase; therefore Vab will increase in the positive direction. The graph in question 2 should now show V_{ab} increasing positively as variable resistance increases.

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